

# In a nutshell: Jacobi's method

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Given a system of  $n$  linear equations in  $n$  unknowns  $A\mathbf{u} = \mathbf{v}$ , we will use iteration to approximate a solution to this system of linear equations. We will assume that  $A$  is strictly diagonally dominant, in which case, we are assured that all the diagonal entries are non-zero.

Parameters:

- $\mathcal{E}_{\text{step}}$     The maximum step size allowed before we consider the method to have converged.
- $N$          The maximum number of iterations.

1. Define  $A_{\text{diag}}$  to be the  $n \times n$  matrix of the diagonal entries of  $A$  and calculate the inverse  $A_{\text{diag}}^{-1}$  of this matrix, which is that matrix with the reciprocals of each of the diagonal entries of  $A_{\text{diag}}$ .
2. Define  $A_{\text{off}}$  to be the  $n \times n$  matrix of the off-diagonal entries of  $A$ .
3. Let  $\mathbf{u}_0 \leftarrow A_{\text{diag}}^{-1} \mathbf{v}$  and  $k \leftarrow 0$ .
4. If  $k > N$ , we have iterated  $N$  times, so stop and return signalling a failure to converge.
5. Set  $\mathbf{u}_{k+1} \leftarrow A_{\text{diag}}^{-1} (\mathbf{v} - A_{\text{off}} \mathbf{u}_k)$ .
6. If  $\|\mathbf{u}_{k+1} - \mathbf{u}_k\|_2 < \mathcal{E}_{\text{step}}$ , return  $\mathbf{u}_{k+1}$ .
7. Increment  $k$  and return to Step 2.

Note that if  $A$  is a sparse matrix (most entries are zero and stored using a sparse-matrix representation), then it is reasonable to calculate  $A_{\text{diag}}^{-1} A_{\text{off}}$  first and then replace Step 5 by:

5'. Set  $\mathbf{u}_{k+1} \leftarrow \mathbf{u}_0 - (A_{\text{diag}}^{-1} A_{\text{off}}) \mathbf{u}_k$ .